

Lecture Notes

On

Time Value of Money

MBA – 2nd sem

BBA – 2nd sem

Subject – Financial Management

By : Dr Monisha Gupta

Department of Management, NGB (DU)

Time Value of Money:

The value of money received today is different from the value of money received after some time in the future. An important financial principle is that the value of money is time dependent.

This principle is based on the following four reasons:

Inflation:

Under inflationary conditions the value of money, expressed in terms of its purchasing power over goods and services, declines.

Risk:

Re. 1 now is certain, whereas Re. 1 receivable tomorrow is less certain. This 'bird-in-the-hand' principle is extremely important in investment appraisal.

Personal Consumption Preference:

Many individuals have a strong preference for immediate rather than delayed consumption. The promise of a bowl of rice next week counts for little to the starving man.

Investment Opportunities:

Money like any other desirable commodity has a price, given the choice of Rs. 100 now or the same amount in one year's time, it is always preferable to take the Rs. 100 now because it could be invested over the next year at (say) 18% interest rate to produce Rs. 118 at the end of one year. If 18% is the best risk-free return available, then you would be indifferent to receiving Rs. 100 now or Rs. 118 in one year's time. Expressed another way, the present value of Rs. 118 receivable one year hence is Rs. 100

Present Value:

It is a method of assessing the worth of an investment by inverting the compounding process to give present value of future cash flows. This process is called 'discounting'.

The present value of 'P' of the amount 'A' due at the end of 'n' conversion periods at the rate 'i' per conversion period.

The value of 'P' is obtained by solving the following equation:

$$P = A (1 + i)^n$$

Example 1 :

Ascertain the present value of an amount of Rs. 8,000 deposited now in a commercial bank for a period of 6 years at 12% rate of interest.

Solution:

$$P = A/(1 + i)^n$$

$$8,000 = A/(1 + i)^n$$

$$8,000 = A/(1 + 0.12)^6$$

$$8,000 = A/1.97382$$

$$A = 8,000 \times 1.97382 = \text{Rs. } 15,791$$

Example 2:

Find out the present value of Rs. 10,000 to be required after 4 years if the interest rate is 6%.

Solution:

$$P = \frac{A}{(1 + i)^n} = \frac{10,000}{(1 + 0.06)^4} = \frac{10,000}{1.26428} = \text{Rs. } 7,921$$

An amount Rs. 7,921 to be deposited into bank to get Rs. 10,000 at the end of 4 years at interest rate of 6%.

Calculation of Discount Factors:

The exercise involved in calculating the present value is known as 'discounting' and the factors by which we have multiplied the cash flows are known as the 'discount factors'.

The discount factor is given by the following expression:

$$\left[\frac{1}{(1 + i)^n} \right]$$

Where 'i' is the rate of interest per annum and 'n' is the number of years over which we are discounting.

Discounted cash-flow is an evaluation of the future cash-flows generated by a capital project, by discounting them to their present day value. The discounting technique converts cash inflows and outflows for different years into their respective values at the same point of time, allows for the time value of money.

Example 3 :

A firm can invest Rs. 10,000 in a project with a life of three years. The projected cash inflow are as follows:

Year	1	2	3
Cash inflows (Rs.)	4,000	5,000	4,000

The cost of capital is 10% p.a. Should the investment be made?

Solution:

Firstly the discount factors can be calculated based on Re. 1 received in with 'i' rate of interest in 3years.

$$\begin{aligned} \text{Year 1} &= \frac{1}{(1 + i)^1} = \frac{1}{(1 + 10/100)} = \frac{1}{(1.10)} = 0.909 \\ \text{Year 2} &= \frac{1}{(1 + 10/100)^2} = \frac{1}{(1.10)^2} = 0.826 \\ \text{Year 3} &= \frac{1}{(1 + 10/100)^3} = \frac{1}{(1.10)^3} = 0.751 \end{aligned}$$

Year	Cash flow (Rs.)	Discount factor	Present value (Rs.)
0	(10,000)	1,000	(10,000)
1	4,000	0.909	3,636
2	5,000	0.826	4,130
3	4,000	0.751	3,004
			NPV = 770

Since the net present value is positive (10000 – 10770 = 770), investment in the project can be made.

The present value of future cashflow can also be ascertained as follows:

$$\begin{aligned} V &= \frac{I_1}{(1 + i)} + \frac{I_2}{(1 + i)^2} + \frac{I_3}{(1 + i)^3} + \dots + \frac{I_n}{(1 + i)^n} \\ &= \frac{4,000}{(1 + 0.10)} + \frac{5,000}{(1 + 0.10)^2} + \frac{4,000}{(1 + 0.10)^3} \\ &= \frac{4,000}{1.10} + \frac{5,000}{1.21} + \frac{4,000}{1.331} \\ &= 3,636 + 4,132 + 3,005 = \text{Rs. } 10,773 \\ \text{NPV} &= \text{Rs. } 10,773 - \text{Rs. } 10,000 = \text{Rs. } 773 \end{aligned}$$

Compounding Rate and Capitalising Rate -The compounding rate is used in project evaluation to determine the present value of past investment / cashflow, whereas the capitalising rate is applied in the reverse process of discriminating present value of future cash flows. Both considers the time value of money.

Annuity:

An annuity is a cashflow, either income or outgoings, involving the same sum in each period. An annuity is the payment or receipt of equal cashflows per period for a specified amount of time. For example, when a company set aside a fixed sum each year to meet a future obligation, it is using annuity.

The time period between two successive payments is called 'payment period' or 'rent period'. The word 'annuity' is broader in sense, which includes payments which can be annual, semiannual, quarterly or any other fixed length of time. Annuity does not necessarily mean payment taken to be one year.

Future Value of Ordinary Annuity – An ordinary annuity is one in which the payments or receipts occur at the end of each period. In a five year ordinary annuity, the last payment is made at the end of the fifth year.

$$A = P \left[\frac{(1 + i)^n - 1}{i} \right]$$

Where,

A = Annual or future value which is the sum of the compound amounts of all payments

P = Amount of each installment

i = Interest rate per period

n = Number of periods

Example 4 :

Mr. X is depositing Rs. 2,000 in a recurring bank deposit which pays 9% p.a. compounded interest. How much amount Mr. X will get at the end of 5th year.

Solution:

$$\begin{aligned} A &= \frac{P}{i} [(1 + i)^n - 1] \\ &= \frac{2,000}{0.09} [(1 + 0.09)^5 - 1] \\ &= \frac{2,000}{0.09} (1.53862 - 1) \\ &= 22,222.22 \times .53862 = \text{Rs. } 11,969 \end{aligned}$$

Present Value of Perpetuity:

Perpetuity is a financial instrument that promises to pay an equal cash flow per period forever, that is, an infinite series of payments and principal amount never be repaid.

The present value of perpetuity is calculated with the following formula:

$$V = P/i$$

Example 5:

X Ltd. had taken a freehold land for an annual rent of Rs. 1,200. Find out the present value of freehold land which is enjoyable in perpetuity if the interest rate is 8% p.a.

Solution:

$$V = \frac{P}{i} = \frac{1,200}{0.08} = \text{Rs. } 15,000 \quad P = \text{Annual rent} \quad i = 0.08$$

Amortisation:

Amortisation is the gradual and systematic writing off of an asset or an account over a period. The amount on which amortisation is provided is referred to as 'amortizable amount. Depreciation accounting is form of amortisation applied to depreciable assets. Depletion is a form of amortisation in case of wasting assets.

The gradual repayment or redemption of loan or debentures is also referred to as amortisation. Sinking fund method and Insurance policy method are used for systematic writing-off of an asset or redemption of bonds and other long-term debt instruments. Present value of an annuity interest factors can be used to solve a loan amortisation problem, where the objective is to determine the payments necessary to pay off or amortise a loan.

Sinking Fund:

It is a kind of reserve by which a provision is made to reduce a liability, e.g., redemption of debentures or repayment of a loan. A sinking fund is a form of specific reserve set aside for the redemption of a long-term debt. The main purpose of creating a sinking fund is to have a certain sum of money accumulated for a future date by setting aside a certain sum of money every year.

It is a kind of specific reserve. Whatever the object or the method of creating such a reserve may be, every year a certain sum of money is invested in such a way that with compound interest, the exact amount to wipe off the liability or replace the wasting asset or to meet the loss, will be available. The amount to be invested every year can be known from the compound interest annuity tables.

Alternatively, an endowment policy may be taken out which matures on the date when the amount required will be paid by the insurance company.

The advantage of this method is that a definite amount will be available while in the case of investment of funds in securities then exact amount may not be available on account of fall in the value of securities. After the liability is redeemed, the sinking fund is no longer required and as it is the undistributed profit, it may be distributed to the shareholders or may be transferred to the General Reserve Account.

Example 6:

A machine costs Rs. 3,00,000 and its effective life is estimated to be 6 years. A sinking fund is created for replacing the machine at the end of its effective life time when its scrap realizes a sum of Rs. 20,000 only. Calculate to the nearest hundreds of rupees, the amount which should be provided, every year, for the sinking if it accumulates at 8% p.a. compounded annually.

Solution:

For accumulation in sinking fund at compound rate we have:

$$A = P/i [(1+i)^n - 1]$$

$$A = 3,00,000 - 20,000 = 2,80,000$$

$$i = 0.08$$

$$n = 6$$

$$2,80,000 = P/0.08 [(1+0.08)^6 - 1]$$

$$2,80,000 = P/0.08 [(1.08)^6 - 1]$$

$$2,80,000 = P/0.08 (1.586874 - 1)$$

$$2,80,000 = P/0.08 \times 0.586874$$

$$2,80,000 = P \times 7.33593$$

$$P = 2,80,000/7.33593 = \text{Rs. } 38,168$$

Term Structure and Interest Rates:

Interest Rates:

The interest rate is an important consideration for a modern finance manager in taking investment and finance decisions. Interest rates are the measure of cost of borrowing. The interest rates of a country will also influence the foreign exchange value of its own currency. Interest rates are taken as a guide in making investments into shares, debentures, deposits, real estates, loan lending etc.

The interest rates differ in different market segments due to the following reasons:

(a) Risk:

Borrowers carrying high risk will pay higher rates of interest than the borrowers with less risk.

(b) Size of Loan:

The higher amounts of deposits carry higher interest than small deposits.

(c) Profit on Re-Lending:

Financial intermediaries make their profits from re-lending at a higher rate of interest than the cost of their borrowing.

(d) Type of Financial Asset:

Different types of financial assets attract different types of interest. For example deposit in a public sector bank carries interest rate of 10%, but a deposit in a private sector company may attract an interest rate of 15%.

(e) International Interest Rates:

The rate of interest may vary from country to country due to differing rates of inflation, Government policies and regulations, foreign exchange rates etc.

Nominal and Real Rates of Interest:

The nominal rates of interest are the actual rates of interest paid. The real rates of interest are the rates of interest adjusted for the inflation. The real rate is, therefore, a measure of the increase in the real wealth, expressed in terms of buying power, of the investor or lender.

The real rate of interest is calculated as follows:

Real rate of interest = $1 + \text{Nominal rate of interest} / 1 + \text{Rate of inflation} - 1$

Example 7:

The nominal rate of interest is 12% and the rate of inflation is 5%. What is the real rate of interest?

Solution:

$$\text{Real rate of interest} = 1 + 0.12 / 1 + 0.05 - 1 = 1.12 / 1.052 - 1 = 1.067 - 1 = 0.067$$

$$\text{Real rate of interest} = 6.7\%$$

Interest Rates, Capital Gains and Losses:

The increase or decrease in the value of stock is calculated as follows:

$$\text{Real value of stock} = \text{Face value of stock} \times \frac{\text{Nominal rate of stock}}{\text{Market Nominal rate}}$$

Example 8:

The long-term gilts issued by the Government with a face value of Rs. 100 and the coupon rate is 10%.

Calculate the resale value of gilts in the following situations:

(a) If the market nominal rate rises to 15%:

$$\text{Resale value of stock} = \text{Rs. } 100 \times \frac{10\%}{15\%} = \text{Rs. } 66.67$$

If the investor sells his stock we will incur a capital loss of Rs. 33.33 (i.e. Rs. 100 – Rs. 66.67)

(b) If the Market nominal rate falls to 7%:

$$\text{Resale value of stock} = \text{Rs. } 100 \times \frac{10\%}{7\%} = \text{Rs. } 142.86$$

If the investor sells his stock he will get a capital gain of Rs. 42.86 (i.e. Rs. 100 – Rs. 142.86)

Interest Rates and Share Prices:

The shares and debt instruments are alternative ways of investment. If the interest rates on debt instruments fall, shares become more attractive to buy. As demand for shares increases, their prices rise too, and so the dividend return gained from them fall in percentage terms. If interest rates went up, the shareholder would probably want a higher return from his shares and share prices would fall.

Changes in Interest Rates and Financing Decisions:

The changes in interest rates will have strong impact on financing decisions taken by a Finance manager.

Financial strategy to be followed when interest rates are low:

- (i) Borrow more moneys at fixed rate of interest to increase the company's gearing and to maximize return on equity.
- (ii) Borrow long-term funds rather than short-term funds.
- (iii) Replace the high cost debt with low cost debt.

Financial strategy to be followed when interest rates are higher:

- (a) Raise funds by issue of equity shares and to stay away from raising debt finances.
- (b) Debt finance can be taken for short-term rather than long-term.
- (c) Surplus liquid assets can profitably be invested by switching of investments from equity shares to interest bearing investments.
- (d) Reduce the need to borrow funds by selling unwanted and inefficient assets, keep the stocks and debtors balances at lower levels etc.
- (e) New projects need to be given careful consideration, which must be able to earn the increased cost of financing the projects.